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# METHOD OF COMPUTING SYSTEM TRACKING MARGINS

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———— GODDARD SPACE FLIGHT CENTER ————  
GREENBELT, MARYLAND

**A METHOD OF COMPUTING  
SYSTEM TRACKING MARGINS**

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#### SUMMARY

This document describes a mathematical computational procedure that can be used to determine the signal level as a function of time for any particular communication or tracking system. Nominal values are assumed for equipment operating characteristics and the range equation is used for free-space attenuation.

The Introduction (Section 1) presents a fundamental discussion of the need for System Tracking Margin determination. It includes a block diagram of the overall system tracking margin determination.

Section 2 defines the aspect angles " $\theta$ " and " $\phi$ " with respect to the spacecraft axes and the spacecraft-tracking station line-of-sight.

Section 3 describes the coordinate systems and transformations necessary to determine the aspect angles. Euler angle rotations are used to describe the transformations between coordinate systems.

Section 4 presents the nominal system characteristics for the Unified S-band and C-band communications and tracking and develops the equations for determining the signal levels.

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## 1. INTRODUCTION

### 1.1 PURPOSE

The purpose of this paper is to describe a mathematical procedure suitable for programming on a 7094 computer that can be used to determine the signal levels as a function of time for a particular piece of communicating or tracking equipment and a particular mission.

For example:

It might be required to know if the Signal-to-Noise ratio to the S-band signal will fall below a certain value for a given site for a given mission. This mathematical procedure would therefore allow a tracking analysis before the mission, thus predicting signal dropouts and/or poor performance areas.

### 1.2 PREPARATION

Figure I is a block diagram of the major elements of the System Tracking Margins Computation. It is assumed that the spacecraft position, spacecraft orientation (pitch, yaw, roll), spacecraft antenna pattern and system characteristics are available as inputs.

It is first necessary to ascertain the vehicle position as a function of time from the given set of orbital elements. This is necessary in order to establish the vector between the vehicle and the ground station from which (ultimately) the aspect angles will be determined. Once the aspect angles have been computed, a table lookup of the supplied spacecraft antenna patterns is performed; and from there, the range equation and the system parameters are used to determine the receiver signal level.

### 1.3 SCOPE

This document will cover only that portion of the System Tracking Margins Computation that accepts vehicle position and outputs signal level as a function of time (See Figure 2). The computation of vehicle position from orbital elements or state vectors is well known and can be obtained from many sources. To facilitate the discussion, the remainder of this document has been divided into the following areas:

- a. Aspect angle calculation
- b. Coordinate systems and their transformation
- c. Signal level computation
  - (1) S-band
  - (2) C-band

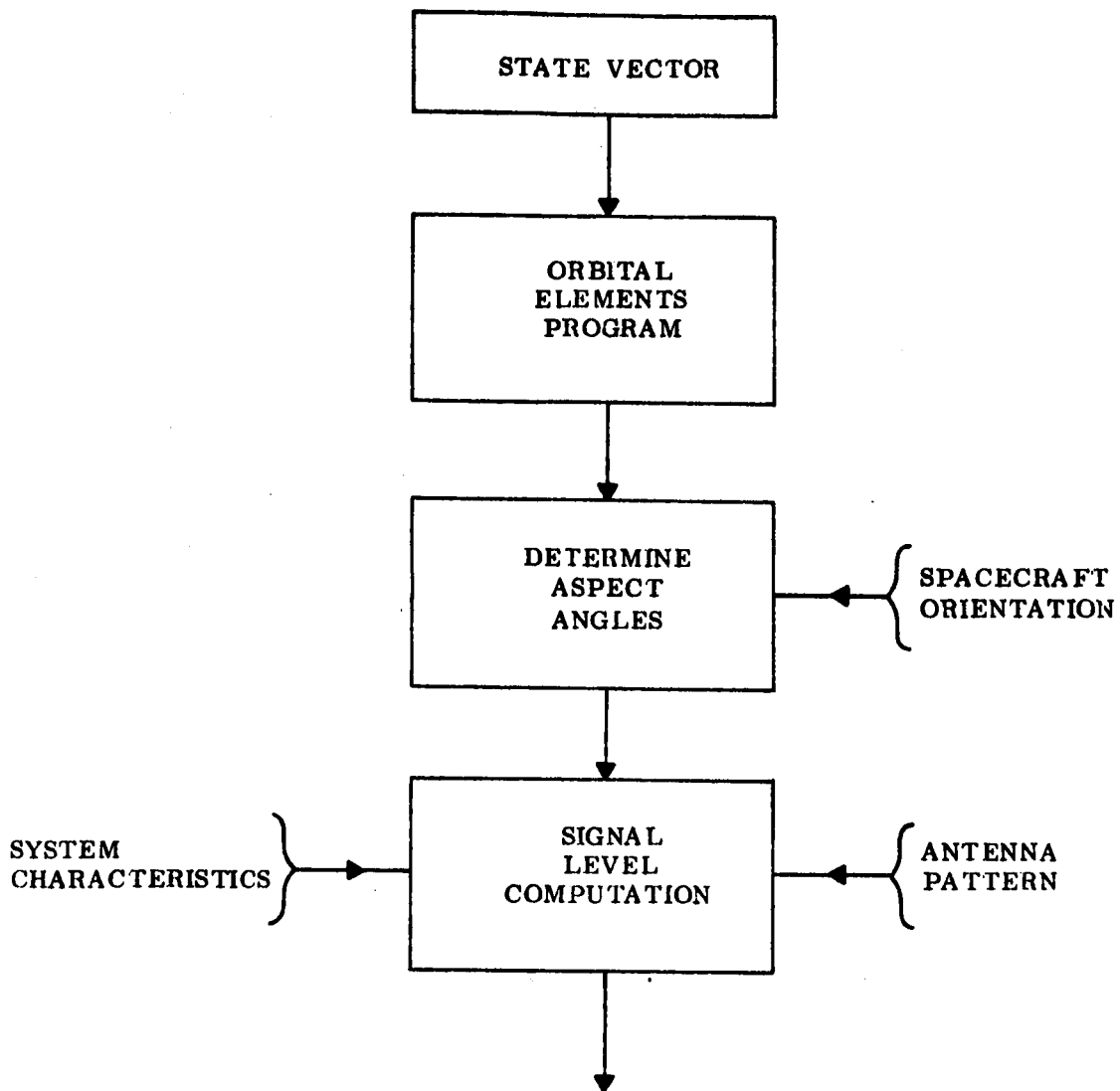


Figure 1. System Tracking Margins Computation

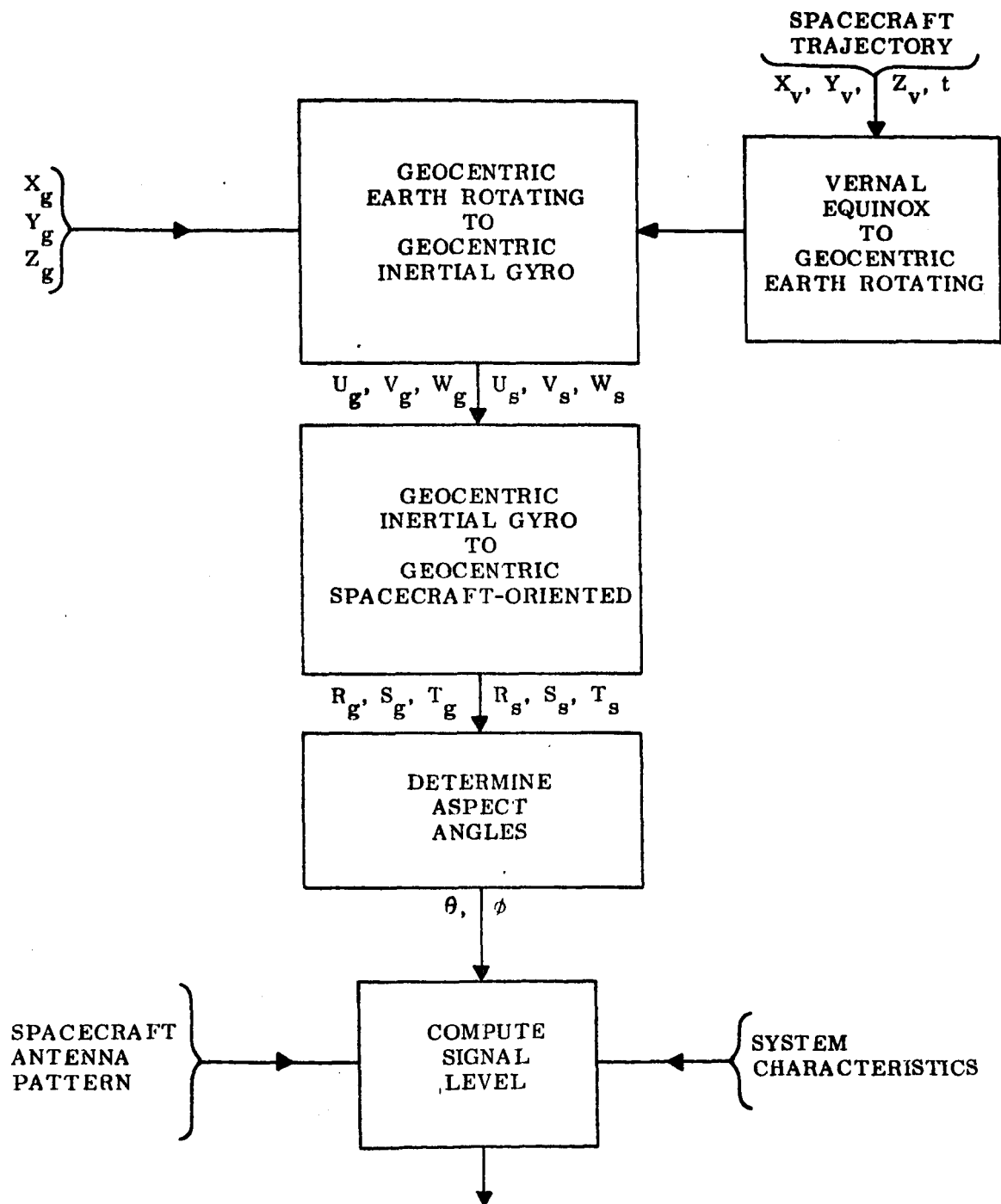


Figure 2. Math Model Block Diagram

## 2. ASPECT ANGLES

### 2.1 DETERMINING ASPECT ANGLES

The aspect angles to be determined are theta ( $\theta$ ) and phi ( $\phi$ ). These two angles uniquely define the attitude of the spacecraft (vehicle) axes with respect to the spacecraft-radar line of sight.

- The aspect angle "theta ( $\theta$ )" is defined as the angle between the spacecraft positive roll axis (R) and the spacecraft-radar line of sight (L) (Figure 3).
- The aspect angle "phi ( $\phi$ )" is defined as the angle measured from the spacecraft negative yaw axis (-T) to the projection of the spacecraft radar line-of-sight in the spacecraft roll plane (Figure 3).

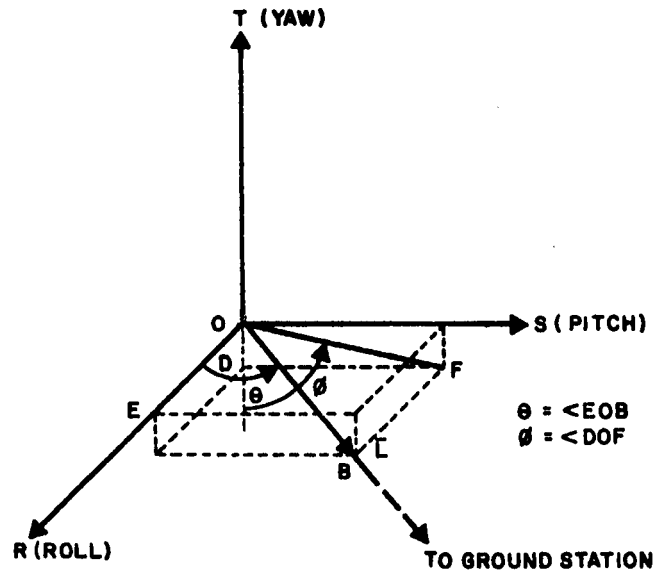


Figure 3. - Aspect Angles Defined

### 2.2 DETERMINING SPACECRAFT-RADAR LINE OF SIGHT VECTOR

The spacecraft-radar line of sight vector " $\vec{L}$ " may be obtained by subtracting the vehicle position from the station position. Both positions are known in a common coordinate system whose axes are parallel to the spacecraft pitch, yaw and roll axes (Figure 3, Equations 1, 2, 3, 4).

The process of obtaining the spacecraft and station positions is explained in Section 3.

$$L_x = R_g - R_s \quad (1)$$

$$L_y = S_g - S_s \quad (2)$$

$$L_z = T_g - T_s \quad (3)$$

$$|L| = \sqrt{L_x^2 + L_y^2 + L_z^2} \quad (4)$$



From figure 3 and using basic trigonometric relations, it may be shown that:

$$\theta = \cos^{-1} \frac{\bar{L}_x}{|\bar{L}|} \quad (5)$$

$$\phi = \tan^{-1} \frac{\bar{L}_y}{-\bar{L}_z} \quad (6)$$

### 3. COORDINATE SYSTEMS AND TRANSFORMATIONS

#### 3.1 GENERAL

All coordinate systems used in defining the station position and the spacecraft position are orthogonal, right-handed, with positive rotations in a counter-clockwise direction as viewed from the positive end of the axis as it is rotated.

In order to determine aspect angles, the spacecraft position and orientation (pitch, yaw, roll) and the station position must be known with respect to a common reference system, e.g., either the station and spacecraft position in terms of the spacecraft axes or in terms of the station axes. Since the aspect angles are defined with respect to the spacecraft axes (pitch, yaw, roll) and angle magnitudes are invariant with translation, a geocentric spacecraft-oriented coordinate system (R, S, T) was selected.

#### 3.2 PROCEDURE

3.2.1 Transform the initial spacecraft and station positions shown in the earth rotating coordinate system (figure 4A) to the inertial gyro-oriented coordinate system (figure 4B) using Euler rotations of the X, Y, Z axes into the U, V, W axes.

3.2.2 Transform the spacecraft and station position from the U, V, W coordinate system to the R, S, T, coordinate system using Euler rotations of the U, V, W axes into the R, S, T axes.

3.2.3. The following transformations will be performed on the spacecraft and station positions:

- a. Geocentric Earth Rotating (X, Y, Z) to  
Geocentric Inertial Gyro (U, V, W)
- b. Geocentric Inertial Gyro (U, V, W) to  
Geocentric Spacecraft-Oriented (R, S, T)

3.2.4 It can be seen from figure 2 that the same transformation equations apply to both the station and vehicle position coordinate transformations.

3.2.5 The station and spacecraft positions are given in a geocentric coordinate system (figure 4A), and the pitch, yaw and roll angles are defined with respect to the gyro axes orientation at lift-off; thus in order to redefine the station and spacecraft position coordinates with respect to a geocentric spacecraft-oriented coordinate system we must first know the launch pad position and the orientation of the gyro axes on the launch pad. The launch pad spherical coordinates are given by its latitude ( $\alpha$ ), longitude ( $\beta$ ). The orientation of the gyro axes is: "U" is pointing downrange (launch azimuth), "W" is the gravity vector (local vertical), and "V" is perpendicular to "U" and "W" so as to form a right-handed coordinate system. (See Figure 4B).

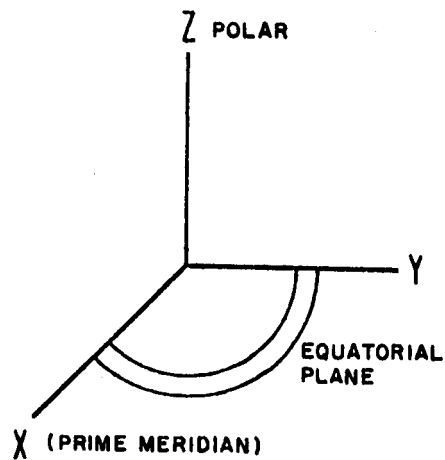


Figure 4A. Geocentric Earth Rotating

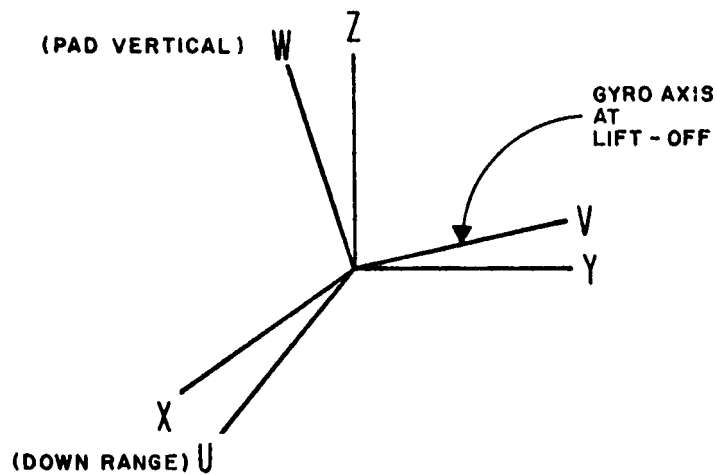


Figure 4B. Geocentric Inertial Gyro

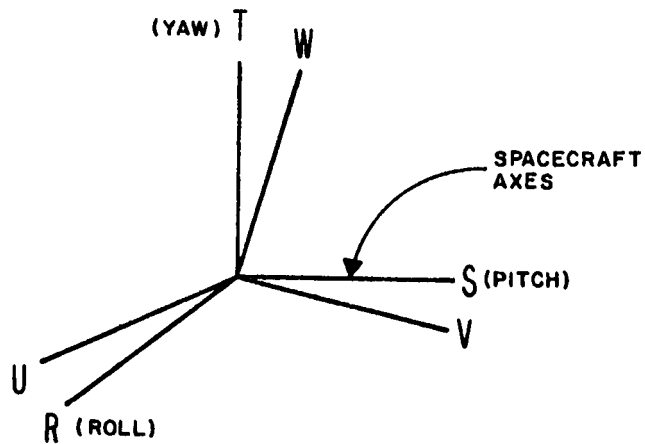


Figure 4C. Geocentric Inertial, Spacecraft Oriented

Figure 4. Geocentric Coordinate Systems

3.2.6 Now, to account for the difference between an earth rotating and an inertial coordinate system, we must rotate the earth by the angle determined by the difference between the time under consideration and lift-off time. Since we are using an equatorial coordinate system, the amount of earth rotation can be directly subtracted from the longitude of the pad to obtain the position of the launch pad at lift-off. Thus, the station and vehicle position may now be defined in the geocentric gyro-inertial system by the Euler Angle matrix equation given below:

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = R_z'' (180 - \gamma) R_y' (90 - \alpha) R_z (B - \omega T) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (7)$$

Where:

$$R_z'' = \begin{pmatrix} \cos A & \sin A & 0 \\ -\sin A & \cos A & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad A = 180 - \gamma_p$$

$$R_y' = \begin{pmatrix} \cos B & 0 & -\sin B \\ 0 & 1 & 0 \\ \sin B & 0 & \cos B \end{pmatrix}; \quad B = 90 - \alpha_p'$$

$$R_z = \begin{pmatrix} \cos C & \sin C & 0 \\ -\sin C & \cos C & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad C = B_p - \omega t$$

We may determine the station and spacecraft position coordinates with respect to the actual pitch (R), roll (S) and yaw (T) axes of the spacecraft by first defining the angles of "pitch", "yaw" and "roll," and then writing the Euler matrix equation. The pitch angle (p) is defined as a counter clockwise rotation about the "V" axis. The yaw angle (y) is defined as a counter clockwise rotation about the "W" axis. The roll angle (r) is defined as a counterclockwise rotation about the "U" axis. Thus the Euler matrix equation can be written as:

$$\begin{bmatrix} R \\ S \\ T \end{bmatrix} = R_v(r) R_w(+y) R_v(+p) \begin{bmatrix} U \\ V \\ W \end{bmatrix} \quad (8)$$

where:

$$R_v''(r) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos r & \sin r \\ 0 & -\sin r & \cos r \end{pmatrix}; \quad r = \text{roll angle}$$

$$R_w'(+y) = \begin{pmatrix} \cos y & \sin y & 0 \\ -\sin y & \cos y & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad y = \text{yaw angle}$$

$$R_v(+p) = \begin{pmatrix} \cos p & 0 & -\sin p \\ 0 & 1 & 0 \\ +\sin p & 0 & \cos p \end{pmatrix}; \quad p = \text{pitch angle}$$

Now, the spacecraft - station line-of-sight vector may be computed using (1), (2), (3), (4) and "theta and phi" calculated by (5) and (6).

#### 4. SIGNAL LEVEL COMPUTATION

##### 4.1 UNIFIED S-BAND

##### 4.1.1 30-FOOT ANTENNA TOTAL RECEIVED POWER

Now that the aspect angles have been computed, the spacecraft antenna gain can be determined from a table lookup of the antenna pattern. Using the Unified S-band system parameters (table I) and calculating the free-space propagation loss,<sup>1</sup> as a function of slant range, the following equation was used to determine the total received signal level for the spacecraft to ground station commo link for the 30-foot ground antenna:

Table 1. - USB 30-foot Antenna, System Characteristics

##### Spacecraft

Transmit Power	40.6dbm (11.8 w)
Transmit Losses	5db
Transmit Frequency	2287.5
Antenna Gain	$G(\theta, \phi)$

##### Ground Station

Antenna Gain	43db
Receiver Losses	2db

$$P_{30} = P_T - L_D + G(\theta, \phi) + 2(\lambda) - 2(R) + G_R - L_R - 127 \quad (9)$$

Where:

$P_{30}$	- Total received power (dbm)
$P_T$	- Spacecraft transmitter power (dbm)
$L_B$	- Spacecraft transmitter losses (db)
$G(\theta, \phi)$	- Spacecraft antenna gain (db)
$\lambda$	- Transmit wavelength (dbcm)
$R$	- Slant range (dbnm)
$G_R$	- Receiver antenna gain (db)
$L_R$	- Receiver losses (db)
127	- Units conversion constant (db)

Substituting the known parameters in (9) we get:

$$P_{30} = 26.8 + G(\theta, \phi) - 20 \log R \quad (10)$$

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1. Barton, David K, "Radar System Analysis." Prentice-Hall, Inc., 1964, p 119

#### 4.1.2 85-FOOT ANTENNA TOTAL RECEIVED POWER

The USB System characteristics for the 85-foot ground antenna system are given in Table 2 and are summarized as follows in equation (11) below:

Table 2. - USB 85-foot Antenna, System Characteristics

##### Spacecraft

$$P_T = 40.6 \text{ dbm}$$

$$L_T = 9\text{db}$$

$$F_T = 2287.5\text{Mc}$$

$$G = 14, 20.5, 26.5\text{db}$$

##### Ground Station

$$G_R = 52\text{db}$$

$$L_R = 2\text{db}$$

$$P_{85} = 21.8 + G - 20 \log R \quad (11)$$

#### 4.1.3 USB DOWNLINK SIGNAL POWER

In order to determine the actual signal power in each of the signals comprising the USB downlink spectrum, the modulation loss of each type of signal (carrier, voice, telemetry, ranging, etc) for each mode of operation must be known. The various signal component modulation losses are tabulated in "db below the total received power" ( $P_{30}$ ,  $P_{85}$ ) in Table 3.<sup>2,3</sup>

Table 3. - Modulation Losses (Downlink)

<u>Mode</u>	<u>Carrier</u>	<u>PRN</u>	<u>TLM</u>	<u>Normal Voice</u>	<u>Backup Voice</u>	<u>Emergency Key</u>
1	4.6	-	5.1(HS)	10.1	-	-
2	4.7	18.6	5.3(HS)	10.3	-	-
3	4.7	18.6	10.3(LS)	4.3	-	-
4	4.6	-	10.1(LS)	4.2	-	-
5	6.8	-	1.9(LS)	-	-	-
6	2.3	-	-	-	-	7.1
7	1.1	6.4	-	-	-	-
8	3.9	-	4.6(L)	-	8.9	-
9	7.0	20.9	2.1(L)	-	-	-
HS = 51.2 kbps				LS = 1.6 kbps		

2. Extracted from CSC-0-058, APL, 25 Jan 6c, p22

3. These MOD losses may vary from mission to mission.

#### 4.1.4 DETERMINING SIGNAL COMPONENT LEVELS

a. In order to determine the individual signal component levels, we modify equations (10) or (11) with the modulation loss "M". This results in the following equations:

$$(1) P(CXR) = P_{30} - M_{cxr} \quad (12)$$

$$(2) P(TLM) = P_{30} - M_{tlm} \quad (13)$$

$$(3) P(VOICE) = P_{30} - M_{voice} \quad (14)$$

$$(4) P(RANGING) = P_{30} - M_{ranging} \quad (15)$$

b. For "Mode 2" operation with the 30-foot ground antenna, the individual signal level may be written as follows:

$$(1) P(CXR) = P_{30} - 4.6 \quad (16)$$

$$(2) P(TLM) = P_{30} - 5.3 \quad (17)$$

$$(3) P(VOICE) = P_{30} - 10.3 \quad (18)$$

$$(4) P(RANGING) = P_{30} - 18.6 \quad (19)$$

c. The above treatment also applies to the 85-foot ground antenna resulting in:

$$(1) P'(CXR) = P_{85} - 4.6 \quad (20)$$

$$(2) P'(TLM) = P_{85} - 5.3 \quad (21)$$

$$(3) P'(VOICE) = P_{85} - 10.3 \quad (22)$$

$$(4) P'(RANGING) = P_{85} - 18.6 \quad (23)$$

#### 4.2 C-BAND

The C-band signal levels for the CSM and the ground station can be computed for the uplink and downlink case using the parameter in Table 4, in conjunction with the previously determined aspect angles and free space propagation.

Table 4. - C-Band System Parameters

##### a. CSM

SENS. = -70dbm

$P_b$  = 2500 watts

$L_b$  = 7 db (AVE.)

$F_d$  = 5765 Mc

b. Ground Station (FPS-16, with paramps)

$$P = 1\text{Mw}$$

$$G = 44\text{db}$$

$$F_u = 5690\text{Mc}$$

$$L_t = 3\text{db}$$

$$L_r = 1.5\text{db}$$

$$B_n = 1.6\text{Mc}$$

$$NF = 4\text{db}$$

c. Ground Station (FPQ-6, with paramps)

$$P = 3\text{Mw}$$

$$G = 51\text{db}$$

$$L_t = 3\text{db}$$

$$L_r = 1.5\text{db}$$

$$F_u = 5690\text{Mc}$$

$$B_n = 1.6\text{Mc}$$

$$NF = 4\text{db}$$

4.2.1 FPS-16 SIGNAL LEVELS

a. Using equation (9) and the FPS-16 parameters in Table 4, the "uplink" signal level can be written as:

$$P_{16}^U = 11.3 + G(\theta, \phi) - 20 \log R \quad (24)$$

b. and the downlink signal level as:

$$P_{16}^D = -13.2 + G(\theta, \phi) - 20 \log R \quad (25)$$

4.2.2 FPQ-6 SIGNAL LEVELS

a. For the FPQ-6 (with paramps), the "uplink" signal level may be represented by:

$$P_6^U = 23.1 + G(\theta, \phi) - 20 \log R \quad (26)$$

b. and the "downlink" signal level as:

$$P_6^D = -2.2 + G(\theta, \phi) - 20 \log R \quad (27)$$

## 5. CONCLUSION

The preceding equations have been programmed on the IBM 7094 computer, and are presently being used to predict the signal levels for the Manned Space Flight Tracking Network Unified-S-Band communication system and C-Band radar tracking system for the Apollo missions.



# APPENDIX

## LIST OF SYMBOLS

$\alpha$	Station latitude (+ north, -south)
$\beta$	Station longitude (+ east, -west)
$\phi$	Angle between the spacecraft negative yaw axis and the projection of the spacecraft line-of-sight vector in the roll plane (deg.)
$\theta$	Angle between the spacecraft roll axis and spacecraft-station line of sight vector (deg).
$\omega$	Angular velocity of the earth (deg/sec)
$\bar{L}$	Spacecraft-station line-of-sight vector
M	Modulation loss
p	Spacecraft pitch angle (deg)
P <sub>30</sub>	Received signal power for 30-foot antenna system (dbm)
P <sub>85</sub>	Received signal power for 85-foot antenna system (dbm)
P(CXR)	Received carrier signal power for 30-foot antenna system (dbm)
P'(CXR)	Received carrier signal power for 85-foot antenna system (dbm)
P(TLM)	Received telemetry signal power for 30-foot antenna system (dbm)
P'(TLM)	Received telemetry signal power for 85-foot antenna system (dbm)
P(VOICE)	Received voice signal power for 30-foot antenna system (dbm)
P'(VOICE)	Received voice signal power for 85-foot antenna system (dbm)
P(RANGING)	Received ranging signal power for 30-foot antenna system (dbm)
P'(RANGING)	Received ranging signal power for 85-foot antenna system (dbm)
P <sub>6</sub> <sup>U</sup>	Uplink received signal level for FPQ-6 (dbm)
P <sub>6</sub> <sup>D</sup>	Downlink received signal level for FPQ-6 (dbm)
P <sub>16</sub> <sup>U</sup>	Uplink received signal level for FPS-16 (dbm)
P <sub>16</sub> <sup>D</sup>	Downlink received signal level for FPS-16 (dbm)
r	Spacecraft roll angle (deg)
R, S, T	General Geocentric Spacecraft-Oriented Inertial Coordinate System ("R" is the roll axis, "S" is the pitch axis, and "T" is the yaw axis).
R <sub>G</sub> , S <sub>G</sub> , T <sub>G</sub>	<u>Station position</u> in a Geocentric Spacecraft-Oriented Inertial Coordinate System.

$R_s, S_s, T_s$	<u>Spacecraft position</u> in a Geocentric Spacecraft Oriented Inertial Coordinate System.
$R_x (\gamma)$	Rotation of " $\gamma$ " degrees about the "X" axis.
T	Ground elapsed time (sec)
U, V, W	General Geocentric Gyro Inertial Coordinate System ("U" pointing Downrange, "W" through pad vertical).
$U_G, V_G, W_G$	<u>Station position</u> in a Geocentric Gyro Inertial Coordinate System
$U_s, V_s, W_s$	<u>Spacecraft position</u> in a Geocentric Gyro Inertial Coordinate System
X, Y, Z	General Geocentric Equatorial Earth-Rotating Coordinate System ("X" through prime meridan, "Z" through polar axis)
$X_G, Y_G, Z_G$	<u>Station position</u> in a Geocentric Earth-Rotating Coordinate System
$X_s, Y_s, Z_s$	<u>Spacecraft position</u> in a Geocentric Earth-Rotating Coordinate System
y	Spacecraft yaw angle (deg)